NATURAL CONVECTION OF AIR IN A RECTANGULAR CAVITY WITH PARTIALLY HEATED AND COOLED SIDE WALLS

Ramonu O.J*, Afolabi S.I, Akinyemi T.O

Abstract— Natural convection of air in a two-dimensional rectangular cavity has been studied numerically using a Galerkin weighted residual finite element method. In the present study, the top and bottom walls are considered adiabatic while the left vertical wall is maintained at constant high temperature T_h with the remaining parts of the left wall considered adiabatic. The cold right vertical wall is maintained at a constant low temperature T_c with the remaining parts of the right-vertical wall considered adiabatic. The pressure-velocity form of the Navier–Stokes equations and energy equation are used to represent the mass, momentum, and energy conservations of the fluid medium in the cavity. The Discontinuous Galerkin formulations of the dimensionless governing equations with the associated boundary conditions are solved by a nonlinear coupled system of fluid flow interface for discretization of all the field variables using COMSOL multiphysics 5.0 version. The Rayleigh number is varied from ($Ra = 10^5$ to 10^7) and Prandtl number is taken as 0.71. This study reported the effects of varying Rayleigh numbers and Nusselt numbers on the thermo-fluid characteristics. The results obtained indicates that the Rayleigh number drastically affects the flow profile and heat transfer behavior within the cavity. Moreover, it is concluded that, for low Rayleigh numbers, natural convection reduces and heat transfer by conduction is predominant in the cavity.

Index Terms— Air; COMSOL; Natural Convection; Rectangular Cavity; Side walls

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1. Introduction

Natural convection in closed cavities has generated more interest for its applications in a wide spectrum of engineering processes such as cooling of microelectronic components, solar energy collectors with insulated strips, nuclear reactors safety, heat exchangers, and solar thermal receivers etc. Effective cooling of electronic components has become increasingly important as power dissipation and component density continue to increase substantially with the fast growth of electronic technologies. It is very important that the cooling systems are designed in the most efficient way and the power requirement for the cooling process is drastically reduced.

The difficulties often encountered in developing experimental studies to determine the parameters related to convection has brought about different methods for solving natural convection problems. The increase in computational power and development of numerical algorithms implemented with systems of differential equations which correlate with the thermal and dynamic aspects of the flow and allow for application of particular initial and boundary conditions, has made the numerical solution of these problems become technically and economically feasible.

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Natural convection in cavities have been studied extensively for its application in wide engineering processes. Ostrach [1] presented a comprehensive detailed bibliography on natural convection in cavities. There have been so many numerical studies on natural convections in rectangular cavities of different configurations, Valencia and Frederick [2], Selamet et al. [3], Hasnaoui et al. [4], Papanicolaou and Gopalakrishna [5], Sundstrom and Kimura [6], Hsu and Chen [7], Elsherbiny et al. [8], and Nguyen and Prudhomme [9]. The natural convection in a vertical square cavity cooled from one side and heated from the bottom side was examined by Anderson and Lauiat [10] while November and Nansteel [11] in their own study gave a detailed report that near the bottom wall, the heated fluid layer is attached to the turning corner. Chu et al.[12] presented a numerical and experimental report on two-dimensional laminar natural convection in rectangular cavities to investigate the influence of the dimensions and location of heater, aspect ratio, and boundary conditions on the rectangular cavity. Turan et al, [13] considered a two-dimensional steady-state problem of laminar natural convection in square cavities with heated sidewalls, they gave a comprehensive report that for both Bingham and Newtonian fluids, increasing the Rayleigh number produces an increase in the mean Nusselt number.

Saleh et al. [14] examined natural convection in a nanofluid-filled trapezoidal enclosure, they reported that the rate of heat transfer was enhanced by the acute sloping wall and high concentration Cu nanoparticles while the natural convection in partially heated rectangular enclosures filled with nanofluids were presented comprehensively by Oztop and Abu [15], their result indicates that, at low Reynolds number, the influence of inclination angle on heat transfer enhancement is insignificant.

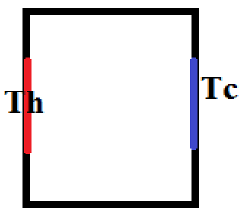


Fig 1: physical geometry of the cavity

Caronna et al, [16] examined natural convection in a square and tall rectangular enclosures filled with air, the problem was subjected to a combination of heating and cooling conditions. These conditions were considered for various height-to-width aspect ratio of the enclosure, the heated fractions of both vertical side walls, and the Rayleigh number. In their own study, Basak et al, [17] considered the effects of linearly heated and cooled vertical side walls with uniformly heated bottom wall on flow and heat transfer characteristics due to mixed convection within a square cavity. They stated that local Nusselt number plot showed that heat transfer rate is equal to 1 at the edges for the case of linearly heated side walls case and it was zero at the left edge and thereafter increased the case of cooled right wall.

The study of natural convection in enclosures can also be applied to nuclear power plant cooling. Water is most commonly used as the working fluid to transfer heat from the reactor core to the steam turbine and to dump the surplus heat from the steam circuit. [18]; In United States of America, a nuclear power plant supplies electricity to about 740,000 homes with 13 gallons of water consumed per household daily in a once-through cooling system. [19] Stated that when the working fluid (water) used for the heat transfer process is returned back to its source, it poses a threat to the environment. Recently, Ramonu and Alagbe [20] numerically analyzed the effects of contamination transport in groundwater by modeling the flow in the soil matrix with the aid of the COMSOL multiphysics software platform. The Water Resources Control Board also confirmed that the once-through cooling systems kill 2.6 million fish a year [21]. The study of natural convection heat transfer process in enclosures could guide researchers to more efficient ways in cooling nuclear power plant while new generations of nuclear power plants are being developed [22].

On the basis of the literature review, it appears that no work was reported on the computational analysis of natural convection of air in a rectangular cavity with partially heated and partially cold sidewalls with adiabatic top and bottom walls, The obtained numerical results are presented graphically and discussed in terms of streamlines, isotherms, local Nusselt number and Rayleigh numbers.

2. Problem Formulation

The motion of a fluid within a partially heated rectangular cavity of length L′ and height H′ is considered in this study (see Fig.1). The fluid within the cavity is assumed to be Newtonian, incompressible and laminar; and the flow field is two-dimensional and in a steady state. All properties of the fluid are considered constant, except the density of the fluid that gives rise to the buoyancy force and varies according to the Boussinesq approximation.

Neglecting viscous dissipation due to small velocities associated with natural convection, the governing equations are written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\vartheta\left\{\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right\}. \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+\vartheta\left\{\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right\}+\beta g(T_h-T_c). \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\propto\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \tag{4}$$

Introducing the stream function, such that

$$u=\frac{\partial\psi}{\partial y},\qquad v=-\frac{\partial\psi}{\partial x}$$

Then, the continuity Eq. (1) is satisfied;

Also, the vorticity equation ω is derived by involving the stream function in terms of u and v.

$$\omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}. \tag{5}$$

Thus, the vorticity transport equation for steady state yields

$$u\left(\frac{\partial\omega}{\partial x}\right)+v\left(\frac{\partial\omega}{\partial y}\right)=\vartheta\left(\frac{\partial^2\omega}{\partial x^2}+\frac{\partial^2\omega}{\partial y^2}\right)+\beta g\frac{\partial T}{\partial x}. \tag{6}$$

The above equations are non-dimensionalized by using the following dimensionless dependent and independent variables

$\Psi=\frac{\psi}{\propto}$, $X=\frac{x}{\delta}$, $\Omega=\frac{\omega\delta^2}{\propto}$,

$Y=\frac{y}{\delta}$, $U=\frac{u\delta}{\propto}$,

$V=\frac{v\delta}{\propto}$, $\theta=\frac{T-T_c}{T_h-T_c}$, $Pr=\frac{\vartheta}{\propto}$,

$Ra=\frac{\beta g(T_h-T_c)\delta^3 Pr}{\vartheta^2}$

Thus, the vorticity transport equation for dimensionless variables is given as:

$$U\frac{\partial\Omega}{\partial X}+V\frac{\partial\Omega}{\partial Y}=Pr\left(\frac{\partial^2\Omega}{\partial X^2}+\frac{\partial^2\Omega}{\partial Y^2}\right)+Ra.Pr\frac{\partial\theta}{\partial X}. \tag{7}$$

The dimensionless form of vorticity equation in terms of stream function

$$\Omega=-\left(\frac{\partial^2\Psi}{\partial X^2}+\frac{\partial^2\Psi}{\partial Y^2}\right). \tag{8}$$

And the dimensionless form of the energy equation is given by

$$U\frac{\partial\theta}{\partial X}+V\frac{\partial\theta}{\partial Y}=\frac{\partial^2\theta}{\partial X^2}+\frac{\partial^2\theta}{\partial Y^2}. \quad (9)$$

And the associated boundary conditions in the dimensionless form:

U = V = 0, the rectangular cavity walls

θ = 0, the cooled active sidewall

θ = 1, the heated active sidewall

$\frac{\partial\theta}{\partial n}$ = 0, the insulated parts of the wall

The following parameters were assumed for the determination of heat transfer behavior.

$\varrho=C_p=k=\mu=1$,

$T_h=1$, $T_c=0$,

Pr=0.71

Ra=10^5 to 10^7

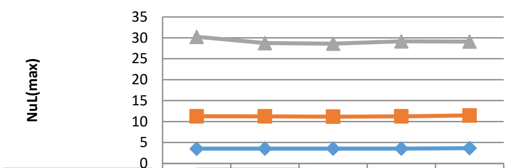
3. Numerical Method

This study focuses on natural convection flow in a rectangular cavity. The property of the rectangular cavity is in 2D, heated partially on its right sidewall and partially cold on the left sidewall with an insulated top and bottom walls. In order to model the problem, the computational modeling and simulation of Numerical problems program, COMSOL Multiphysics software was used. COMSOL Multiphysics software is a powerful finite element, and partial differential equation solution engine. It repeatedly solves problems of buoyant flow in various cavity geometries and analyze different temperature distributions as well as convective flow patterns. The iterative process is tuned for a fast and efficient solution using dimensionless parameters and a Bousinessq approximation for the buoyant force with the Laminar flow and the Heat transfer in fluids interfaces.

3.1 Program Validation and Comparison with Previous works

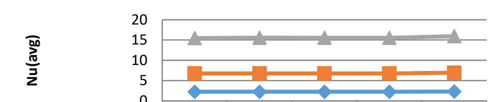
The Computer code was validated with other numerical studies in literatures in order to confirm the accuracy of the numerical procedure employed in this study. However, a good agreement between the present results and the past studies contained in literatures indicates that the simulation of natural convection using COMSOL Multiphysics is an efficient and stable numerical method.

The graphs below show the numerical results that compare the maximum local Nusselt number and average Nusselt number along the hot walls and the locations where they occur. The left wall is heated while the right wall is cooled and the top and bottom wall is kept insulated. Mousa, (2010) modeled laminar buoyancy convection in a square cavity containing an obstacle, while using a penalty finite element method. Barakos et al, (1994) analyzed natural convection flow in a square cavity revisiting laminar and turbulent models with wall functions. Davis, 1983 reported a benchmark numerical solution of natural convection of air in a square cavity and Fusegi et al, (1991) conducted a numerical study of three dimensional natural convection in a differentially heated cubical enclosure. The results obtained from COMSOL Multiphysics simulation provide an excellent agreement with other numerical methods



	Present work	Mousa [2010]	Barakos et al [1994]	Davis [1983]	Fusegi et al [1991]
Ra=10^6	18.981	17.5	17.442	17.925	17.67
Ra=10^5	7.742	7.716	7.636	7.717	7.795
Ra=10^4	3.524	3.531	3.539	3.528	3.652

Figure 1a Validation of Maximum local Nusselt number



	Present work	Mousa [2010]	Barakos et al [1994]	Davis [1983]	Fusegi et al [1991]
Ra=10^6	8.725	8.819	8.806	8.799	9.01
Ra=10^5	4.516	4.521	4.51	4.519	4.646
Ra=10^4	2.242	2.245	2.245	2.243	2.302

Figure 1b: Validation of average local Nusselt number

4. RESULTS AND DISCUSSIONS

Fig. 4a Model geometry

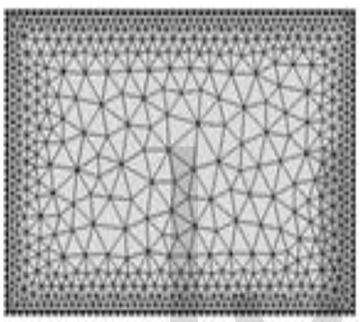


Fig. 4b Mesh generation

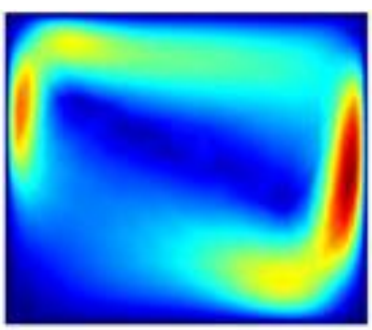


Fig.5a: Velocity profile at Ra=10^5

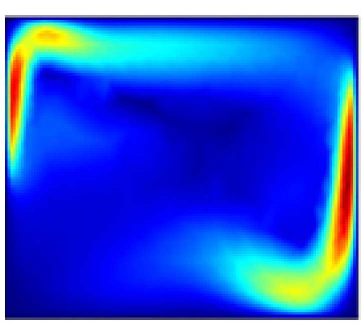


Fig. 5b: Velocity profile at Ra= 10^6

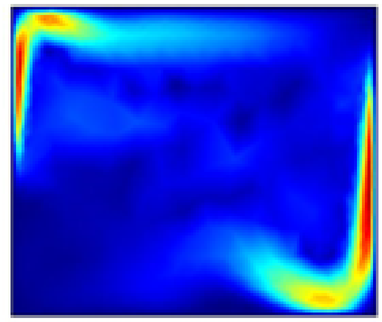


Fig. 5c: Velocity profile at Ra= 10^7

The Velocity profile represent the motion of fluid within the hot and cold regions of the Cavity, at the hot section of the wall, the velocity is observed to increase as the Rayleigh number increases. While at the Cold region of the wall, the velocity is observed to reduce as the Rayleigh number increases, this signifies that conduction reduces within the cavity as convection increases with increases in Rayleigh numbers.

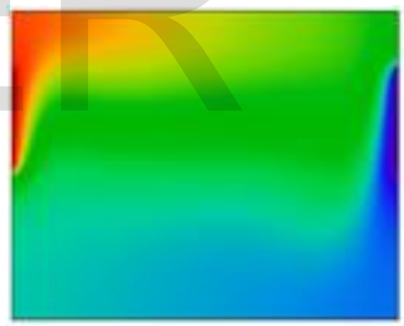


Fig. 6a Temperature Profile Ra=10^5

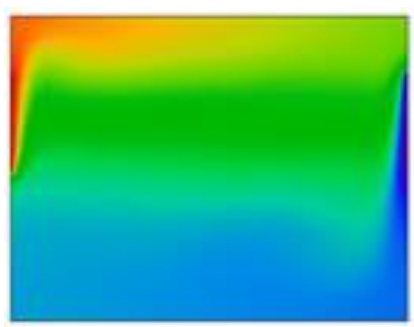


Fig. 6b Temperature Profile Ra=10^6

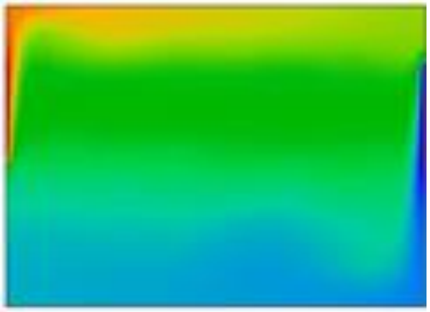


Fig. 6c Temperature Profile Ra=10^7

The temperature profile reveals that at the hot section, the temperature increases with increase in Rayleigh numbers which symbolizes an increase in convection while at the cold region, the temperature reduces as the conduction reduces with increase in convection due to increased Rayleigh numbers

Fig. 7a Isotherm at Ra=10^5.

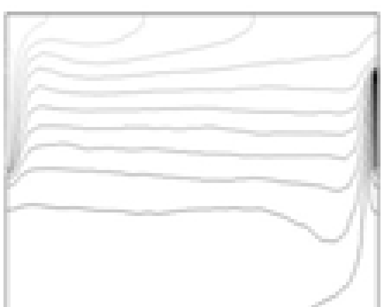


Fig. 7b Isotherm at Ra=10^6

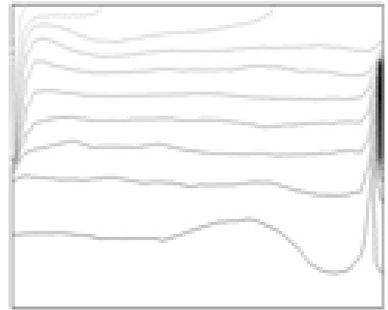


Fig. 7c Isotherm at Ra=10^7

The isotherm plots obtained are almost parallel to the differential heated walls, indicating that the formation of almost horizontal isotherms are due to fact that, heat transferred is by conduction within the two differentially heated walls. As the Rayleigh number increases, the effect of convection seems to be dominant than conduction. The isotherms tend to be horizontally parallel to the wall at the center of cavity, isotherms tend to be horizontal at the center of the cavity and almost vertically parallel when close to the hot and cold regions of the side walls. This situation leads to the thin temperature layers at the region nearest to the vertical hot cold section of the walls. The effect of mostly horizontal isotherms formed at the center of the cavity is due to the dominant effect of convection mode of heat transfer mechanism.

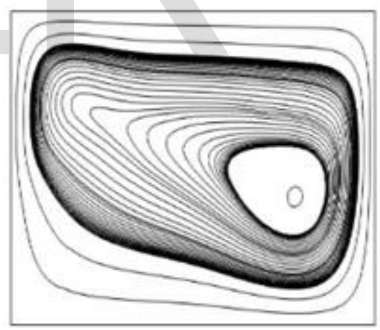


Fig.8a Streamline at Ra=10^5

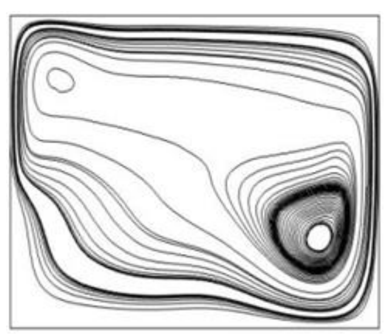


Fig.8b Streamline at Ra=10^6

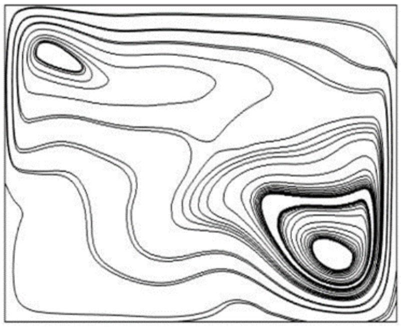


Fig. 8c Streamline at Ra=10^7

Streamlines with circular pattern are formed by a single vortex with its position just below the cold region of the cavity, after undergoing assembling iterations as the Rayleigh increases, the vortex transformed into two, above the hot region and below the cold region, this is the effect of increased convection mode of heat transfer in the cavity due to increased Rayleigh number, The two vortices thereafter becomes more dominant at Ra=10^7. The fluid flow circulation pattern produces two vortices at the Upper left and bottom right of the cavity due to the increase in relative magnitude flow in convective heat transfer.

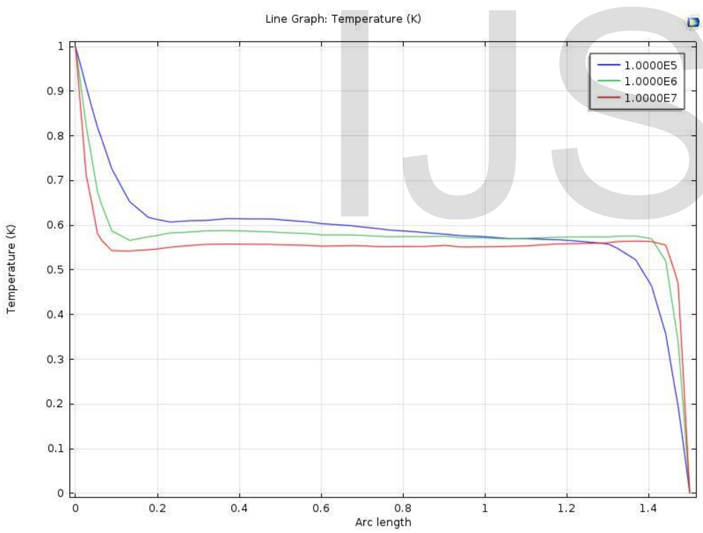


Fig. 9 Graph of the temperature distribution across the cavity at (0, 0.5) (1.5, 0.5)

The graph indicates that the temperature increases at region close to the hot wall and remains constant until it reaches the cold region of the wall where it is then reduced. It is observed that convection increases as the Rayleigh number increases.

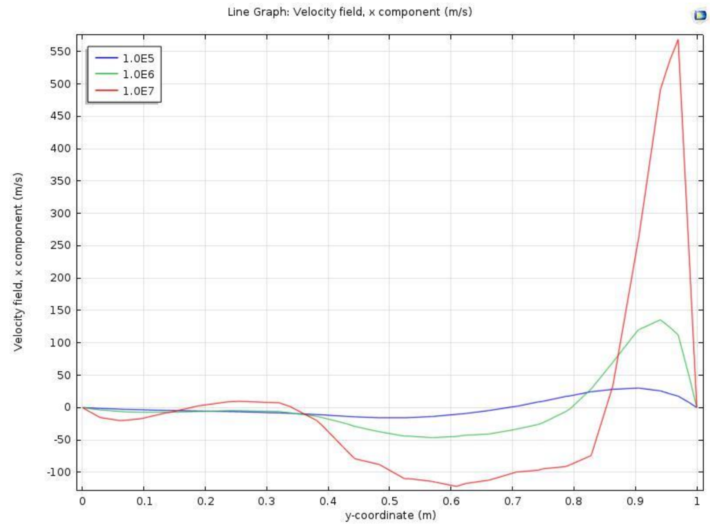


Fig. 10a: Graph showing the velocity distribution

Fig.10b: Graph showing the velocity distribution

The graph shows the velocity distribution along the vertical wall where the heat is applied, Velocity is highest when the Rayleigh was increased for section just above the heater, but lowest at the region of the heater for increased Rayleigh number.

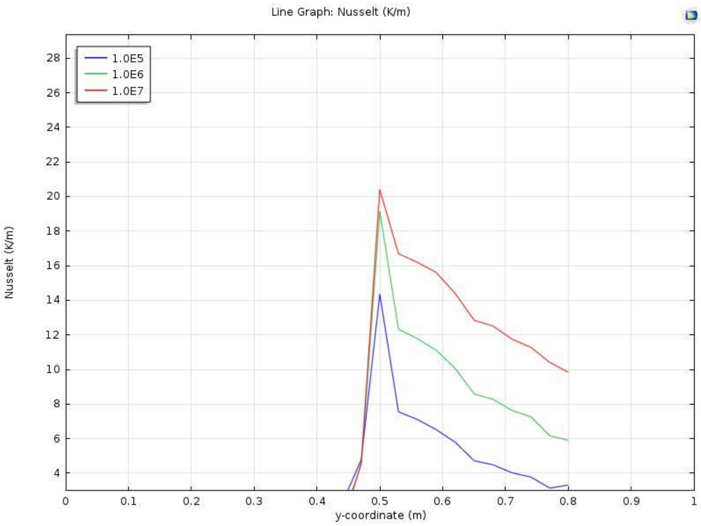


Fig. 11 The graph of Local Nusselt number measured along the only hot region at the right wall

The graph shows that the local Nusselt number reduces upward the length of the wall and increases as the Rayleigh number increases, the resultant of transformation from conduction dominant to convection dominant.

5. Conclusion

The numerical study of natural convection in closed cavity with partially heated left wall and partially cooled right wall has been carried out. Results were obtained in terms of Isotherms, streamlines, temperature distribution and velocity profile. The Rayleigh number drastically affected the flow profile and heat transfer behavior within the cavity, local Nusselt number strongly depends on the Rayleigh number changes. Moreover, for low Rayleigh numbers, natural convection reduces and heat transfer by conduction is experienced in the cavity.

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